Fractional-Order Modelling in Modelica

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Most dynamic systems with a basis in nature can be described using Differential-Algebraic Equations (DAE), and hence be modelled using the modelling language Modelica. However, the concept of DAEs can still be generalized, when differential operators of non-integer order are considered. These so called fractional order systems have counterparts in naturally occurring systems, like electrochemistry and viscoelasticity.

A simple application is given by the relationship between heat flow and temperature at the boundary of a semi-infinite domain. Here, fractional differential operators appear:

$$T(t) = \frac{\alpha^{1/2}}{2 \cdot A \cdot k} \cdot \frac{\delta^{-1/2} Q(t)}{\delta t^{-1/2}} + T_0$$

This paper presents an implementation of approximate fractional-order differential operators in Modelica, increasing the scope of systems that can be described in a meaningful way. Properties of fractional-order systems are discussed and some approximation methods are presented. An implementation in Modelica is proposed for the first time. Several testing procedures and their results are displayed. The work is then illustrated by the application of the model to several physically motivated examples. A possible usability-enhancement using the concept of "Calling Blocks as functions" is suggested.

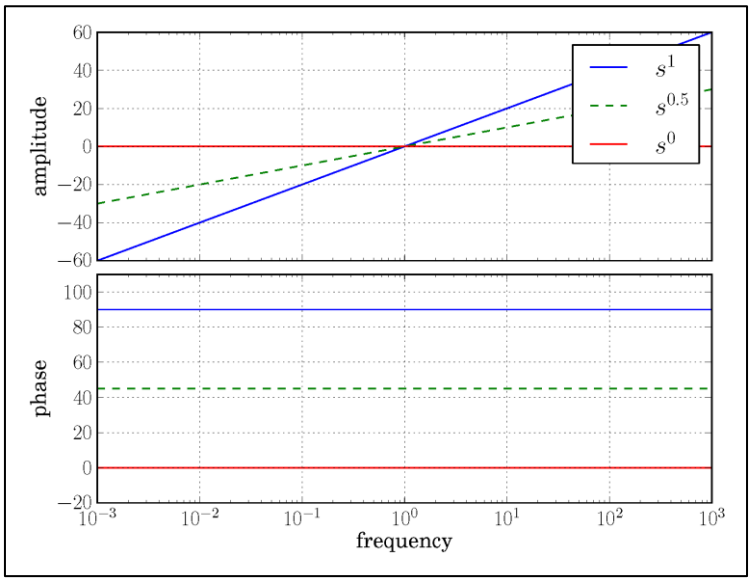

Figure 1. Bodeplots of the unity operator, half-derivator and derivator