

Internalized State-Selection: Generation and Integration of Quasi-Linear Differential-Algebraic Equations

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MODELICA is a language for modeling of dynamical processes. In general, the model equations that describe the dynamical process consist of differential equations in combination with algebraic constraints, i.e., we have to deal with so-called *differential-algebraic equations* (DAEs). The solutions of such systems have to satisfy the algebraic constraints, but, in general, not all constraints are stated in an explicit way. In particular, if the resulting system of DAEs is of higher index there exist so-called *hidden constraints* and the numerical treatment leads to instabilities, inconsistencies and possibly non-convergence of the numerical methods.

We present a novel (i.e. different from the dummy derivative method of Mattsson and Söderlind (1993)) regularization approach for the remodeling of dynamical systems that uses the hidden constraints information provided by the structural analysis, in particular by the Signature Method of Pryce (2001), to construct an over-determined system regularization that can be solved using a specially adapted numerical integrator implemented in the software package QUALIDAES (QUAsi LInear DAE Solver) (see (Steinbrecher, 2006) for details). This integrator is developed for the numerical treatment of *quasi-linear DAEs* of the form

$$E(x,t)\dot{x} = k(x,t) \quad (1)$$

Practical application demands to transform a general MODELICA-style equation into such a quasi-linear form. Naturally, there is a trivial transformation that replaces all derivative with simple identities of the form $\text{DER}(u_i) = u_j$. Since these identities are trivially quasi-linear, this does not violate the requirements for the output of the transformation. However, the resulting system would be unnecessary large and not leverage the structure of the system for efficient simulation. In fact, QUALIDAES would have to solve the whole nonlinear system as hidden constraints.

In this paper we show how such a transformation can be implemented and how the result of this transformation can be differentiated up to an arbitrary degree. This allows for the distinction between transformation and regularization, which is in turn a prerequisite for variable structure modeling and simulation.

References

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